Análise Vetorial

$A = (a_1 ; a_2 ; a_3) \quad B = (b_1 ; b_2 ; b_3)$

$A \times B = -B \times A$

$$A \times B = \begin{vmatrix} i & j & k & i & j \\ a_1 & a_2 & a_3 & a_1 & a_2 \\ b_1 & b_2 & b_3 & b_1 & b_2 \end{vmatrix} =$$

$$(a_2 b_3)\hat{i} + (a_3 b_1)\hat{j} + (a_1 b_2)\hat{k}$$

$$-(a_3 b_2)\hat{i} - (a_1 b_3)\hat{j} - (a_2 b_1)\hat{k}$$

$$-B \times A = \begin{vmatrix} i & j & k & i & j \\ -b_1 & -b_2 & -b_3 & -b_1 & -b_2 \\ a_1 & a_2 & a_3 & a_1 & a_2 \end{vmatrix} =$$

$$-(a_3 b_2)\hat{i} - (a_1 b_3)\hat{j} - (a_2 b_1)\hat{k}$$

$$(a_2 b_3)\hat{i} + (a_3 b_1)\hat{j} + (a_1 b_2)\hat{k}$$

$$A \times B = -B \times A \ //$$

$$(A+B)\times C = A\times C + B\times C$$

$$\begin{vmatrix} i & j & k & i & j \\ a_1+b_1 & a_2+b_2 & a_3+b_3 & a_1+b_1 & a_2+b_2 \\ c_1 & c_2 & c_3 & c_1 & c_2 \end{vmatrix} =$$

$$(a_2c_3+b_2c_3)\hat{i} + (a_3c_1+b_3c_1)\hat{j} + (a_1c_2+b_1c_2)\hat{k}$$
$$-(a_3c_2+b_3c_2)\hat{i} - (a_1c_3+b_1c_3)\hat{j} - (a_2c_1+b_2c_1)\hat{k} =$$

$$(a_2c_3)\hat{i} + (a_3c_1)\hat{j} + (a_1c_2)\hat{k} - (a_3c_2)\hat{i} - (a_1c_3)\hat{j} - (a_2c_1)\hat{k} +$$
$$(b_2c_3)\hat{i} + (b_3c_1)\hat{j} + (b_1c_2)\hat{k} - (b_3c_2)\hat{i} - (b_1c_3)\hat{j} - (b_2c_1)\hat{k} =$$

$$A\times C + B\times C \;//$$

$$A \times (B + C) = A \times B + A \times C$$

$$\begin{vmatrix} i & j & k & i & j \\ a_1 & a_2 & a_3 & a_1 & a_2 \\ c_1+b_1 & c_2+b_2 & c_3+b_3 & c_1+b_1 & c_2+b_2 \end{vmatrix} =$$

$$(a_2c_3 + a_2b_3)\hat{i} + (a_3c_1 + a_3b_1)\hat{j} + (a_1c_2 + a_1b_2)\hat{k}$$
$$-(a_3c_2 + a_3b_2)\hat{i} - (a_1c_3 + a_1b_3)\hat{j} - (a_2c_1 + a_2b_1)\hat{k} =$$

$$(a_2c_3)\hat{i} + (a_3c_1)\hat{j} + (a_1c_2)\hat{k} - (a_3c_2)\hat{i} - (a_1c_3)\hat{j} - (a_2c_1)\hat{k} +$$
$$(a_2b_3)\hat{i} + (a_3b_1)\hat{j} + (a_1b_2)\hat{k} - (a_3b_2)\hat{i} - (a_1b_3)\hat{j} - (a_2b_1)\hat{k} =$$

$$A \times C + A \times B$$

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$$

$$B \times C = \begin{vmatrix} i & j & k & i & j \\ b_1 & b_2 & b_3 & b_1 & b_2 \\ c_1 & c_2 & c_3 & c_1 & c_2 \end{vmatrix} =$$

$$(b_2 c_3)\hat{i} + (b_3 c_1)\hat{j} + (b_1 c_2)\hat{k}$$

$$-(b_3 c_2)\hat{i} - (b_1 c_3)\hat{j} - (b_2 c_1)\hat{k} =$$

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ (b_2 c_3) - (b_3 c_2) & (b_3 c_1) - (b_1 c_3) & (b_1 c_2) - (b_2 c_1) \end{vmatrix} =$$

$$(a_2 c_2 b_1 - a_2 b_2 c_1)\hat{i} + (a_3 c_3 b_2 - a_3 b_3 c_2)\hat{j} + (a_1 c_1 b_3 - a_1 b_1 c_3)\hat{k}$$

$$-(a_3 b_3 c_1 - a_3 c_3 b_1)\hat{i} + (a_1 b_1 c_2 - a_1 c_1 b_2)\hat{j} - (a_2 b_2 c_3 - a_2 c_2 b_3)\hat{k}$$

$$B(A\cdot C) = B(a_1c_1 + a_2c_2 + a_3c_3) = (a_1c_1b_1 + a_2c_2b_1 + a_3c_3b_1)\,\hat{i} \; +$$

$$(a_1c_1b_2 + a_2c_2b_2 + a_3c_3b_2)\,\hat{j} \; +$$

$$(a_1c_1b_3 + a_2c_2b_3 + a_3c_3b_3)\,\hat{k}$$

$$C(A\cdot B) = C(a_1b_1 + a_2b_2 + a_3b_3) = (a_1b_1c_1 + a_2b_2c_1 + a_3b_3c_1)\,\hat{i} \; +$$

$$(a_1b_1c_2 + a_2b_2c_2 + a_3b_3c_2)\,\hat{j} \; +$$

$$(a_1b_1c_3 + a_2b_2c_3 + a_3b_3c_3)\,\hat{k}$$

$$B(A\cdot C) - C(A\cdot B) =$$

$$(\cancel{a_1c_1b_1} + a_2c_2b_1 + a_3c_3b_1 - \cancel{a_1b_1c_1} - a_2b_2c_1 - a_3b_3c_1)\,\hat{i} \; +$$

$$(a_1c_1b_2 + \cancel{a_2c_2b_2} + a_3c_3b_2 - a_1b_1c_2 - \cancel{a_2b_2c_2} - a_3b_3c_2)\,\hat{j} \; +$$

$$(a_1c_1b_3 + a_2c_2b_3 + \cancel{a_3c_3b_3} - a_1b_1c_3 - a_2b_2c_3 - \cancel{a_3b_3c_3})\,\hat{k} \; =$$

$$\left(a_2c_2b_1 + a_3c_3b_1\right)\hat{i} - \left(a_2b_2c_1 + a_3b_3c_1\right)\hat{i} +$$

$$\left(a_1c_1b_2 + a_3c_3b_2\right)\hat{j} - \left(a_1b_1c_2 + a_3b_3c_2\right)\hat{j} +$$

$$\left(a_1c_1b_3 + a_2c_2b_3\right)\hat{k} - \left(a_1b_1c_3 + a_2b_2c_3\right)\hat{k} =$$

$$\left(a_2c_2b_1 - a_2b_2c_1\right)\hat{i} - \left(a_3b_3c_1 - a_3c_3b_1\right)\hat{i} +$$

$$\left(a_1c_1b_2 - a_1b_1c_2\right)\hat{j} - \left(a_3b_3c_2 - a_3c_3b_2\right)\hat{j} +$$

$$\left(a_1c_1b_3 - a_1b_1c_3\right)\hat{k} - \left(a_2b_2c_3 - a_2c_2b_3\right)\hat{k} =$$

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B) \quad //$$

$$\text{rot}(\text{grad}\,\phi) = 0$$

$$\vec{\nabla} \times \left(\frac{\partial\phi}{\partial x} ; \frac{\partial\phi}{\partial y} ; \frac{\partial\phi}{\partial z}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial\phi/\partial x & \partial\phi/\partial y & \partial\phi/\partial z \end{vmatrix}$$

$$\frac{\partial}{\partial y}\frac{\partial\phi}{\partial z}\hat{i} + \frac{\partial}{\partial z}\frac{\partial\phi}{\partial x}\hat{j} + \frac{\partial}{\partial x}\frac{\partial\phi}{\partial y}\hat{k}$$

$$-\frac{\partial}{\partial z}\frac{\partial\phi}{\partial y}\hat{i} - \frac{\partial}{\partial x}\frac{\partial\phi}{\partial z}\hat{j} - \frac{\partial}{\partial y}\frac{\partial\phi}{\partial x}\hat{k} =$$

$$\cancel{\frac{\partial}{\partial y}\frac{\partial\phi}{\partial z}}\hat{i} + \cancel{\frac{\partial}{\partial z}\frac{\partial\phi}{\partial x}}\hat{j} + \cancel{\frac{\partial}{\partial x}\frac{\partial\phi}{\partial y}}\hat{k}$$

$$-\cancel{\frac{\partial}{\partial y}\frac{\partial\phi}{\partial z}}\hat{i} - \cancel{\frac{\partial}{\partial z}\frac{\partial\phi}{\partial x}}\hat{j} - \cancel{\frac{\partial}{\partial x}\frac{\partial\phi}{\partial y}}\hat{k} = 0$$

Conservação de Massa

1) $0,5 = \frac{1,5}{2} V_{min} \times 0,1 + 1,0 v_{min} \times 0,1$

$0,5 = 0,075 \, v_{min} + 0,1 \, v_{min}$

$v_{min} = 2,8571$

$v_{max} = 2,5 v_{min} = 7,1428$

2) $0,1 = 0,015 \times 0,15 \, v_3$

$v_3 = 0,1 \; m/s$

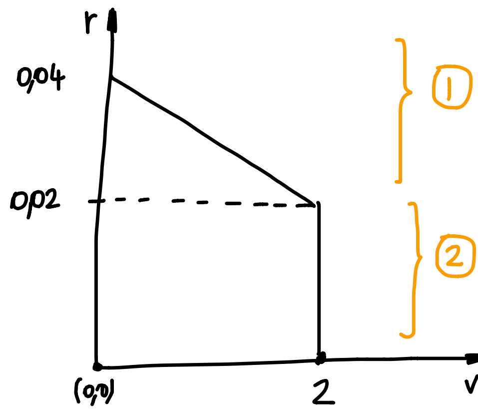
3) $UR^2 = \frac{V_{max}}{2} R^2$

$U = 1,5 \; m/s$

4) $\int_x v_y \, dx = \frac{U\delta}{3}$

$\int_x v_y \, dx = \frac{U}{3} c x^{1/2} \longrightarrow v_y = \frac{U}{6} c x^{-1/2}$

8)



$v_z = \frac{0{,}04 - r}{0{,}01}$

$v_z = 2$

$\dot{m}_{total} = \dot{m}_1 + \dot{m}_2$

$\dot{m}_{total} = 6{,}663 + 2{,}50$

$m_{total} = 9{,}163 \ \frac{kg}{s}$

Conservação de Quantidade de Movimento

1) $\vec{\nabla} \cdot \vec{v} = 0$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial (e^{-2y} \cos x)}{\partial y} = 0$$

$$v_x = \int 2 e^{2y} \cos x \, dx$$

$$v_x = 2e^{2y}\sin x + c(y)$$

Para calcular o gradiente de pressão utiliza-se Navier-Stokes substituindo v_x e v_y na equação.

2) $\vec{\nabla}\cdot\vec{v} = 0$ (incompressível)

$\vec{v} = (Ax+B)\,\hat{i} - Ay\,\hat{j} \longrightarrow \vec{v}$ tem componentes i e j → 2D

aceleração $= \dfrac{D\vec{v}}{Dt}$

→ conferindo se o escoamento é realmente incompressível.

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$\frac{\partial (Ax+B)}{\partial x} + \frac{\partial(-Ay)}{\partial y} = 0$$

$A - A = 0$ ✓ é incompressível!

$$\frac{D\vec{v}}{Dt} = \begin{bmatrix} A^2x \\ A^2y \end{bmatrix} \longrightarrow \frac{D\vec{v}(1,2)}{Dt} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Para o cálculo de $\vec{\nabla} p$, usamos as Eq. de Navier-Stokes:

$$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho}\vec{\nabla} p + \nu \nabla^2 \vec{v} + \vec{g}$$

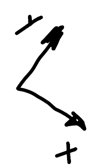
→ pois o campo de velocidades é um dado do problema

$v_x = Ax + By$ e $v_y = -Ay$

5) vazão é o fluxo de massa: $\int_S \rho v_j n_j \, dA$

fluxo de Quantidade de Mov.: $\int_S \rho v_i v_j n_j \, dA$

Como só há escoamento em x



$$\int_S \rho v_x n_x \, dA \longrightarrow \text{fluxo de massa}$$

$$\int_S \rho v_x v_x n_x \, dA \longrightarrow \text{fluxo de Q.M.}$$

Bastando substituir v_x nos fluxos e encontrar a resposta.

Não precisa calcular o perfil de velocidades para μ variável!

b) $\nu_{água} = 8,40 \times 10^{-7}$

$\nu_{ar} = 2,144 \times 10^{-5}$

$\nu_{óleo} = 4,17 \times 10^{-5}$

$\nu_{etileno} = 3,16 \times 10^{-6}$

$$Re = \frac{Ud}{\nu}$$

$$\boxed{U_{crítico} = \frac{\nu \, Re_{crítico}}{d}}$$

→ bastando aplicar a fórmula para diversos fluidos com 2 diâmetros diferentes d_1 e d_2, obtendo assim Re_1 e Re_2